

SHENTON COLLEGE

Examination Semester One 2018 Question/Answer Booklet

MATHEMATICS SPECIALIST UNIT 3

Section One (Calculator-free)

Your name

Time allowed for this sectionReading time before commencing work: 5 minutesWorking time for paper:50 minutes

Material required/recommended for this section

To be provided by the supervisor Question/answer booklet for Section One. Formula sheet.

To be provided by the candidate

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid/tape, ruler, highlighters Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this examination

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	8	8	50	53	35
Section Two: Calculator-assumed	13	13	100	97	65
			Total	150	100

Instructions to candidates

The rules for the conduct of Western Australian external examinations are detailed in the Year 12 *Information Handbook 2018.* Sitting this examination implies that you agree to abide by these rules

Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: if you need to use the space to continue an answer, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number.
 Fill in the number of the question(s) that you are continuing to answer at the top of the page.

Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than 2 marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

It is recommended that you do not use pencil, except in diagrams.

QUESTION	MARKS AVAILABLE	MARKS AWARDED
1	6	
2	7	
3	7	
4	6	
5	10	
6	5	
7	6	
8	6	
TOTAL	53	

STRUCTURE OF THIS PAPER

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Section One: Calculator-free

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1

(6 marks)

The graph of y = f(x) is shown below, where *f* is defined by $f(x) = \frac{5}{1 + \sqrt{x - 3}}$.



(a) Sketch the graph of $y = f^{-1}(x)$ on the same axes.

(2 marks)

(b) Determine the defining rule for $y = f^{-1}(x)$ and state its domain. (4 marks)

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Quest	tion 2		(7 marks)
Consi	der $f(z) = 5z^3 + 2z^2 + 10z + 4$, where z	z is a complex number.	
(a)	Determine, with reasons, which of the	following are factors of $f(z)$.	

(ii) $z - \sqrt{2}i$. (2 marks)

(b) Solve the equation f(z) = 0.

(3 marks)

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Question 3

(7 marks)

(a) Locate the roots of the complex equation $z^5 - 1 = 0$ in the Argand plane below. (3 marks)



- (b) State the sum of all the roots of the complex equation $z^5 1 = 0$. (1 mark)
- (c) Let u be any 5th root of unity, where Im $u \neq 0$.

Show that
$$(1 + u)^2(1 + u^3) = 1 + u + u^4$$
. (3 marks)

Question 4

- (a) Solve this system of equations.
- x + y + 2z = 1 4x + y - z = 73x - y + z = 14

- (b) Determine the value of constant *a* so that the following system of equations does not have a unique solution and give a brief geometric interpretation of the system of equations with this value. (3 marks)

$$x + y + 2z = 1$$

$$4x + y - z = 7$$

$$ax - y + z = 14$$

(6 marks)

(3 marks)

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CALCULATOR-FREE

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Question 5

(10 marks)

The points A, B and C have position vectors (2, 1, -1), (-1, b, -2) and (1, -1, 0) respectively.

(a) Determine the vector equation for the line through *A* and *C*. (2 marks)

(b) Determine, in terms of *b*, the Cartesian equation of the plane containing *A*, *B* and *C*. (5 marks)

(c) The line with equation $\mathbf{r} = (-2, -2, 1) + \mu(p, -2, 7)$ is perpendicular to the plane containing *A*, *B* and *C*. Determine the values of the constants *b* and *p*. (3 marks)

Question 6

The complex numbers u and v satisfy the equations u - v = 2i and uv = 10.

Solve the equations for *u* and *v*, giving your solution(s) in the form x + yi, where *x* and *y* are real.

(6 marks)

Question 7





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(6 marks)

(1 mark)

Question 8

A function is defined by $f(x) = \frac{5-x}{(5x+8)(2x-3)}$.

(a) State the natural domain of f(x).

(b) State the equations of all asymptotes of the graph of $y = x \cdot f(x)$. (2 marks)

(c) The graph of $y = \frac{1}{f(x)}$ has an asymptote with equation y = ax + b. Determine the values of the constants *a* and *b*. (3 marks)

Supplementary page

Question number: _____